MODIFIED SUBSTITUTE STRUCTURE METHOD FOR ANALYSIS OF EXISTING BUILDINGS

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SYNOPSIS

This paper is concerned with the earthquake hazard evaluation of buildings constructed before the most recent advances in seismic design codes. A simplified, linear method is presented for predicting the behaviour, including inelastic response, of existing reinforced concrete structures with known properties and strengths, when subjected to a given type and intensity of earthquake motion, as represented by a linear response spectrum. The technique involves an extension of the Shibata and Sozen substitute-structure method, which was originally proposed as a design procedure. It computes ductility demand of the existing members via an elastic modal analysis, in which reduced stiffness and substitute damping factors are used iteratively. By this means it is possible to describe, in approximate general terms, the location and degree of damage that would occur in an existing building as a result of earthquakes of different intensity. Several reinforced concrete structures of different sizes and strengths were tested by this technique and the results compared with a non-linear time-step analysis. The method appears to work well for structures in which yielding is not extensive and widespread.

RESUME

Cette communication évalue l'effet des séismes sur les bâtiments conçus avant l'existance de normes pour le calcul sismique. Une méthode linéaire simplifiée est présentée pour prévoir le comportement non-linéaire des structures en béton lorsque les propriétés et résistances sont connues, ceci en utilisant évidemment la méthode spectrale pour une intensité déterminée. La méthode est une extension de la technique de Shibata et Sozen qui consiste à évaluer la ductilité des membrures par la méthode linéaire modale et à substituer directement pour une rigidité réduite et des taux d'amortissements différents. Le tout est effectué d'une façon itérative. Plusieurs structures en béton armé ont été évaluées par cette méthode ainsi que par la méthode d'intégration numérique directe. La technique proposée semble prédire des résultats adéquats pour les structures où la plastification n'est pas trop intense.

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INTRODUCTION

In any large city, it is inevitable that many large buildings will have been constructed before the most recent advances in seismic design codes. Performance of these buildings in a major earthquake is at best uncertain; some may survive with only minor damage while others may suffer extensively and even collapse. A first attempt at a comprehensive treatment of seismic hazard evaluation of existing buildings was made by the Applied Technology Council which was formed in the U.S. by several groups associated with the engineering design, supervision and construction of buildings. A screening procedure and a method of analysis for potentially hazardous buildings are outlined in their report, ATC III(1). To check the degree of compliance with current seismic codes it has been proposed that structures be required to resist some fraction of the static loads used for the code design of new structures. This approach has the drawback that older buildings were probably designed with different, if any, ductility requirements and different member detailing from those implied in the current building codes. An ideal procedure would be to subject the structure to a nonlinear dynamic analysis under the most probable seismic ground motion, but the difficulty of modelling inelastic response, the need to consider several ground motions, and the resulting high cost make this procedure impractical for most cases.

This paper proposes an alternate procedure for the analysis of existing reinforced concrete structures. It was developed from a design procedure proposed by Shibata and Sozen (2). Their method called for use of a modified elastic analysis in which the stiffness and damping properties were changed so that the maximum forces and deformations would agree with nonlinear dynamic analysis. Since the structure which is actually analyzed is not the real one, the procedure is called the "substitute structure method". They reported that the location of the plastic hinges was as anticipated and the ductility demands were very much as planned when a structure was designed by their method and then subjected to nonlinear dynamic analysis.

In the design of new structures, ductility and stiffness are known in advance, and required yield level is determined by a single elastic analysis of the substitute structure. In an existing building initial stiffness and yield levels are known and it is required to find the ductility demand. The modified substitute structure method is an iterative procedure in which the stiffness and damping properties of the structure are modified successively until the computed moments agree with the yield moments for all the members which extend into the plastic range. When the iteration procedure is complete, an estimate of ductility demands and floor displacements is obtained. Since member properties and details are known prior to the analysis, it is possible to judge whether each member can withstand the calculated amount of deformation. It is thus possible to describe in general terms the location and extent of damage that might occur in a building under earthquake excitation.

Neither Shibata and Sozen nor the present authors have been able to prove that the method gives the correct solution either for design or analysis; it has been found by trial to give results which are acceptable within practical limits under certain conditions.

The method is described in detail in the first part of the paper. This is followed by presentation of the results obtained by analysis of some test frames, and a comparison with nonlinear dynamic analysis.

Description of the Modified Substitute Structure Method

The design procedure presented by Shibata and Sozen (2) makes use of modified stiffness and damping properties which were derived from dynamic tests on concrete structures (3); since the proposed analysis employs the same computational procedure, its use is presently restricted to reinforced concrete structures. It is possible that with proper modification of the damping properties, the method can be extended to steel structures.

A substitute structure is a hypothetical elastic structure, the stiffness of which is related to but different from the actual frame. Suppose that the moment-rotation relationship of a member in the actual frame can be idealized as a bilinear curve as shown in Fig. 1. If k is the initial stiffness as shown in the figure, and point A represents the maximum moment and rotation reached in the earthquake, then OA defines k_s the stiffness of the substitute member. The damage ratio μ is defined as the ratio of these stiffnesses:

$$\mu = \frac{k}{k_s} \qquad (1)$$

It is emphasized that in the analysis problem, the damage ratio and therefore substitute structure stiffness are not known in advance.

The damage ratio is closely related to member ductility; they are numerically equal for an elastic-perfectly plastic moment-rotation relationship, but the ductility ratio is always greater than the damage ratio corresponding to the same rotation when the material strain hardens. If R is the ratio of the tangent stiffness after yield to the initial stiffness, the relation between the damage ratio, μ and ductility η is given by

$$\mu = \frac{\eta}{1+(\eta-1)R} \qquad (2)$$

The suggested damping ratio for each of the substitute members (2) is given by

$$\beta_s = 0.2(1 - 1/\sqrt{\mu}) + 0.02 \qquad (3)$$

where β_s is a substitute damping ratio and μ is the damage ratio for that member. Equation (3) is based on tests by Gulkan and Sozen (3). A method of computing modal damping ratios is described by Shibata and Sozen (2), where it is assumed that each member contributes to the modal damping in proportion to the relative flexural strain energy associated with each mode shape.

The procedure used in the modified substitute structure method is as follows:

1. Set all damage ratios initally to one.

2. Perform a modal analysis, assuming elastic behaviour and setting damping ratios to values considered appropriate for the given earthquake level. For example, 10% damping may be used for a reinforced concrete structure under a strong earthquake motion. Compute the root-sum-square (RSS) moments.

3. Compare the RSS moments with yield moments and locate the members in which the yield moments are exceeded. Modify the damage ratios for every such member in accordance with

$$\mu_2 = \frac{M_1}{M_p} \qquad (4)$$

where μ_2 = damage ratio for the next iteration.

M_1 = RSS moment from the first iteration.

M_p = plastic moment at the appropriate rotation.

4. Use the new damage ratio to calculate the flexural stiffness of each substitute frame member, EI_s, to be used in the second iteration:

$$EI_s = \frac{EI_a}{\mu_2} \qquad (5)$$

where the EI_a are the initial flexural stiffnesses of the members based on the cracked section. Recompute substitute member damping ratios, modal damping ratios, periods, mode shapes and RSS moments.

5. Repeat steps 3 and 4, modifying the damage ratios as follows:

$$\mu_{n+1} = \mu_n \frac{M_n}{M_p} \qquad (6)$$

where μ_{n+1} = damage ratio for the (n+1)th iteration

μ_n = damage ratio used in the (n)th iteration

The derivation of Eq. (6) is based on Fig. 2, in which it is assumed that the rotation in the (n+1)th iteration will be the same as in the (n)th.

If in any iteration the calculated damage ratio becomes less than unity it indicates that the member has remained elastic in the previous iteration and the new damage ratio is then set equal to unity.

6. Continue to iterate until all the computed moments, except those in members with damage ratios of unity, are sufficiently close to the respective plastic moments. At this stage, the damage ratios have the sought-for values, from which the ductility demands can be deduced.

The following expression is used as a convergence criterion:

$$\left| \frac{M_n - M_p}{M_p} \right| < \varepsilon \qquad (7)$$

Equation (7) is applied to all the members with damage ratios greater than one. If the inequality is satisfied in these members, iteration is stopped. A value of ε equal to 0.001 proved to be satisfactory. It is possible to select other convergence criteria, but this method has worked successfully. Since computational cost is approximately proportional to the number of iterations that is required to satisfy a prescribed convergence criterion, it is highly desirable to increase the rate of convergence. It was found that this could be done by over-correcting the damage ratios at the end of each iteration. Setting

$$\mu'_n = \mu_n + \alpha(\mu_n - \mu_{n-1}) \qquad (8)$$

where μ'_n = over-corrected damage ratio to be used for (n)th iteration.

μ_n = damage ratio computed at the end of (n-1)th iteration according to Eq.(6).

μ_{n-1} = damage ratio used in (n-1)th iteration.

Setting α about 1 proved helpful in reducing the number of iterations when convergence was slow. The over-correction was applied after the first five to ten cycles, and in some cases reduced the number of iterations by a third to a half.

Only a moderate change is necessary to convert an existing modal analysis program to one that can handle the modified substitute structure method. The substitute structure method is incorporated in the subroutine in which modal forces and displacements are computed. The stiffness routine must be changed to handle the modified flexural stiffnesses, but very few changes are required in other parts of the program. The modified substitute structure analysis is more costly than an elastic modal analysis, but its cost is still only a fraction of that of a full nonlinear dynamic analysis. Storage requirements are approximately the same as those in an elastic modal analysis.

Examples and Results

Three test frames will be presented to demonstrate the effectiveness of the modified substitute structure method. Damage ratios and displacements were computed by this method using a smoothed response spectrum. The response histories of the frames were then computed using a nonlinear dynamic analysis program, and the results of the two analyses were compared. The test frames were selected to represent small to medium-sized reinforced concrete structures, but they were not modelled on actual buildings. The choice of member properties and strengths was quite arbitrary; no particular attempt was made to control the amount and location of inelastic deformation. The intention was to test the modified substitute structure method with test frames which might have been designed without consideration of seismic effects. The moment of inertia of the members, intended to represent the cracked section properties, was based on a fraction of that for the gross section.

The smoothed response spectrum used in the substitute structure analysis and the earthquake records used in the nonlinear analysis were employed, among others, by Shibata and Sozen (2). The response spectrum was a smoothed average of six records; namely both components of each of El Centro (1940), Taft (1952) and Managua (1972). The nonlinear dynamic analysis used the two El Centro and the two Taft records but not the Managua records. Shibata and Sozen (2) found that the Managua records produced roughly the same response in a yielding structure as the Taft earthquake and so they were not used here. Figure 3 shows smoothed spectra for 2% and 10% damping for a peak ground acceleration $A_{max} = 0.5$ g. The response acceleration for any damping ratio was related (2) to response at 2% damping in accordance with:

$$\frac{\text{Response acceleration for } \beta}{(\text{Response acceleration for } \beta = 0.02)} = \frac{8}{6+100\beta} \qquad (9)$$

A nonlinear dynamic analysis program for reinforced concrete frames, SAKE (4,5), was used to compute response histories of the three test frames under the four earthquake motions. The reinforced concrete members were modelled by an element with degrading stiffness and with hysteresis rules as described in Ref. (6). Stiffness after yield is assumed to be 2% of the initial stiffness. Stiffness proportional viscous damping, corresponding to a 2% damping ratio in the first mode, was used.

The first example was the 3-bay, 6-storey frame shown in Fig. 4a. Member sizes and stiffness properties for the beams and columns were uniform throughout, but the yield moments varied as shown.

Damage ratios calculated by the modified substitute structure method are shown in Fig. 4b. Average damage ratios, calculated by the nonlinear analysis method for the four earthquakes, are shown in Fig. 4c. The damage ratios for each individual earthquake are shown in Fig. 4d. A value less than unity indicates that the member remained elastic and the numerical value indicates the maximum moment as a proportion of the yield moment. Comparing the modified substitute structure method results with the average of those from the nonlinear analyses (Fig. 4b and Fig. 4c) it is seen that, except for the upper floor beams, the agreement is quite good. There is a general trend for the modified substitute structure method to overestimate the damage ratios in the upper floor beams and underestimate them in the lower floors. Otherwise the values generally fall within the scatter of the results from individual earthquake records.

Table I shows the periods of the initial elastic structure, the substitute structure, and an estimate of the first period from the non-linear analyses. It can be seen that the latter falls between the two former periods, as might be expected. Table II shows the lateral displacements of the floors for the substitute structure and the average results of the non-linear analyses.

The second example was the 3-bay, 3-storey frame of Fig. 5a. The yield moments were such as to make the frame unsymmetric. Damage ratios by the modified substitute structure method are shown in Fig. 5b, and the average values from the four dynamic analyses in Fig. 5c. The results from individual earthquake records appear in Fig. 5d. In this case, there was a wide scatter in the dynamic analyses under different earthquake records, but the agreement between the substitute structure results and the average of the dynamic analyses was excellent.

Table III shows the period comparisons, and Table IV the lateral displacements.

The single-bay, six-storey frame shown in Fig. 6a was the last test structure. The damage ratios obtained by the present method are shown in Fig. 6b, the averages of the dynamic analyses in Fig. 6c, and the individual dynamic values in Fig. 6d. It will be seen that the results from the present method compare badly with those from the dynamic analysis. However, there is also a large scatter in the dynamically calculated response to individual earthquakes. Clearly, erratic behaviour is inherent in this structure, possibly because extensive yield is widespread in both columns and beams, and because it is a single-bay frame. Tables V and VI give the period and displacement comparisons.

Damage ratios did converge, although not always monotonically, in all the structures tested by the authors. The rate of convergence is not easy to predict, but in general the greater the number of yielding members and the higher the final damage ratios, the slower the rate of convergence. Damage ratios of members in the lower stories converge more quickly than those in the upper stories. Each damage ratio changes most rapidly during the first five to ten iterations, with the rate of change decreasing in subsequent iterations.

Once convergence has been achieved, the question remains as to whether the solution is reasonable. However, it should be noted that, once converged, the modified substitute structure analysis is identical to the original substitute structure method as it would be applied to the design of that structure. Thus the validity of the analysis hinges upon that of the design procedure as discussed by Shibata and Sozen (2). They list limitations on the structural systems that can be analyzed, some of which the authors feel that experience may show to be unduly restrictive.

Final Comments

Accuracy of the Method

A procedure has been presented for determining damage ratios in an existing building. These values are necessary for establishing the position of damage and assessing the ability of a structure to resist an earthquake. Obviously these quantities cannot be predicted precisely for uncertain future seismic events. Thus in spite of its imprecision, the method may constitute a useful practical tool.

The method appears to work well for structures in which yielding occurs mainly in the beams. For one of the structures analysed, in which there was extensive yielding in all the beams and columns, the results were bad. However, even a full non-linear dynamic analysis showed this structure behaving erratically under different earthquake records.

It may be noted that the substitute structure as defined must give a longer period than the real structure, and therefore spectral accelerations will be slightly in error. For this reason, other definitions of the substitute structure are under investigation; in particular, an equal energy criterion for definition of the substitute stiffness has led to very good agreement in the period and displacements.

Application of the Method

A rational retrofit procedure should be based upon some estimate of the damage that would be sustained by the building under different levels of seismic activity. Such an estimate cannot readily be based upon an elastic analysis of the ratio of the code lateral force which the structure can carry. Once damage ratios have been obtained, however, an attempt can be made to assess the probable damage, although more research will be needed to relate damage ratios to damage in members that were not properly detailed for seismic resistance.

Thus buildings which are judged, after the first screening, to require analysis may be treated as follows:

1. Determine damage ratios or ductility demands under one or more level of seismic activity.

2. Make certain that these damage ratios can, in fact, be reached: that there is no danger of premature brittle failure due to shear or detailing.

3. Relate the damage ratios to actual damage likely to be sustained.

4. Decide where and how to strengthen the structure.

The present method of analysis is a cheap and effective way of carrying out step 1 above. It is much cheaper than a full non-linear dynamic analysis; we believe it is better than a linear elastic analysis in that it takes account of the redistribution of forces as members begin to yield.

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TABLE 1: 3-bay, 6-storey frame-periods

	Natural Periods in sec.		
Mode	Initial Elastic	Substitute Structure	Non linear Analysis Average
1	1.07	1.66	1.25
2	0.34	0.48	
3	0.19	0.24	
4	0.12	0.14	
5	0.090	0.096	
6	0.075	0.076	

TABLE 2: 3-bay, 6-storey frame-displacements

	Displacements in inches					
Level	El Centro EW	El Centro NS	Taft S69E	Taft N21E	Nonlinear Analysis Average	Substitute Structure
1	1.3	1.1	1.3	0.98	1.2	1.1
2	3.5	2.9	3.1	2.5	3.0	3.0
3	5.9	4.5	4.5	3.7	4.7	5.0
4	7.9	5.5	5.8	4.6	6.0	6.7
5	9.2	6.1	6.6	5.1	6.8	7.9
6	9.8	6.3	7.3	5.4	7.2	8.8

TABLE 3: 3-bay, 3-storey frame-periods

Mode	Natural Periods in sec Initial Elastic	Substitute Structure	Nonlinear Analysis Average
1	0.94	1.22	1.04
2	0.30	0.36	
3	0.14	0.16	

TABLE 4: 3-bay, 3-storey frame-displacements

Level	Displacements in inches El Centro EW	El Centro NS	Taft S69E	Taft N21E	Nonlinear Analysis Average	Substitute Structure
1	3.0	2.4	1.6	1.6	2.2	2.2
2	6.7	5.2	3.0	3.0	4.7	5.0
3	10.6	7.9	5.2	5.2	7.5	8.0

TABLE 5: 1-bay, 6-storey frame-periods

Mode	Natural Periods in sec Initial Elastic	Substitute Structure	Nonlinear Analysis Average
1	1.08	1.85	1.65
2	0.37	0.84	
3	0.21	0.38	
4	0.15	0.28	
5	0.10	0.17	
6	0.077	0.13	

TABLE 6: 1-bay, 6-storey frame-displacements

	Displacements in inches					
Level	El Centro EW	El Centro NS	Taft S69E	Taft N21E	Nonlinear Analysis Average	Substitute Structure
1	3.7	0.74	1.4	2.4	2.1	0.71
2	8.2	1.7	3.3	4.8	4.5	2.1
3	12.0	3.0	4.8	6.1	6.5	2.9
4	14.5	4.5	6.7	6.6	8.1	3.3
5	17.0	6.5	9.4	6.9	10.1	6.8
6	19.3	8.4	11.6	7.2	11.6	8.6

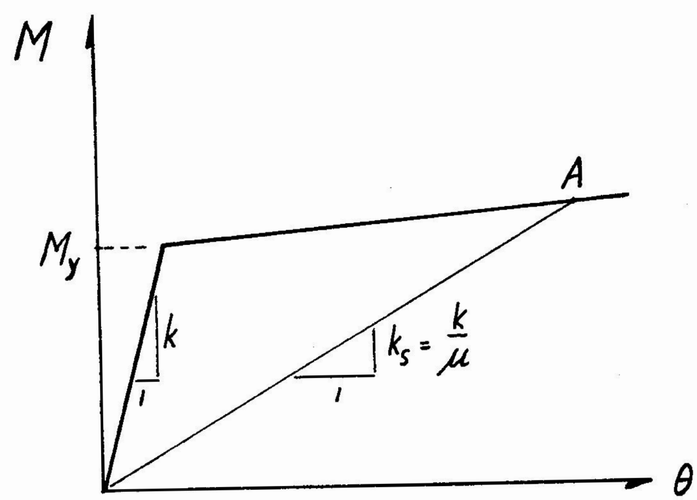


Fig. 1. Moment curvature relationship: stiffness definitions

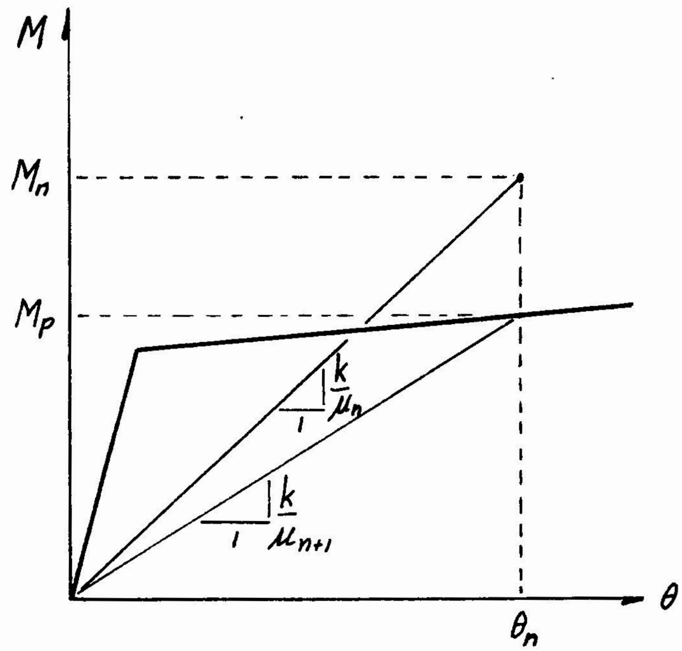


Fig. 2. Moment curvature relationship: damage ratio iterations

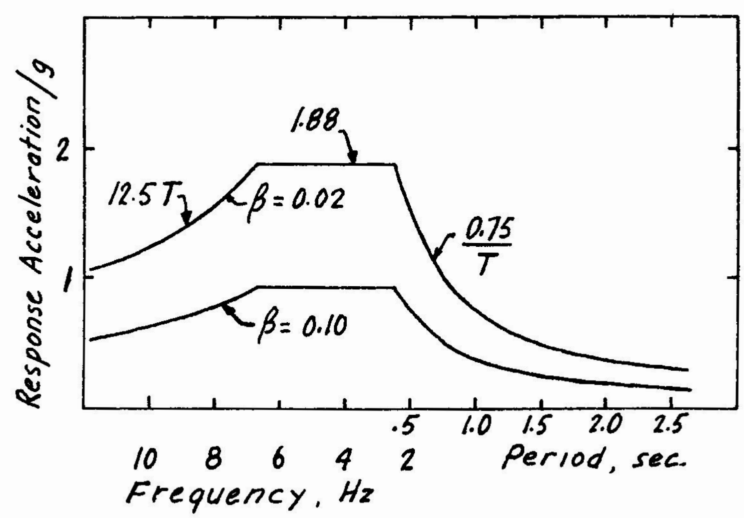


Fig. 3. Acceleration response spectrum: smoothed average of six earthquakes normalised to A_{max} = 0.5g.

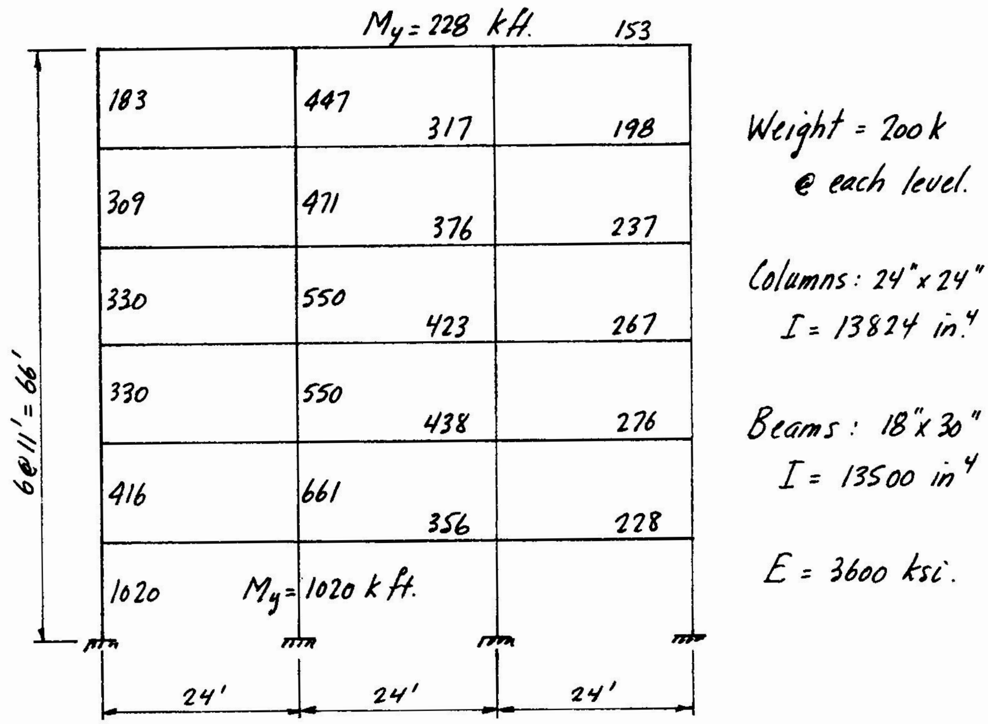


Fig. 4a. Symmetric 3-bay, 6-storey frame: layout, member properties, weight.

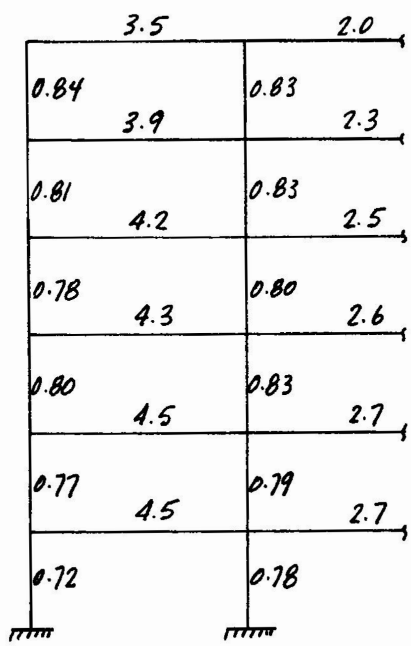


Fig. 4b. Modified substitute structure analysis: damage ratios.

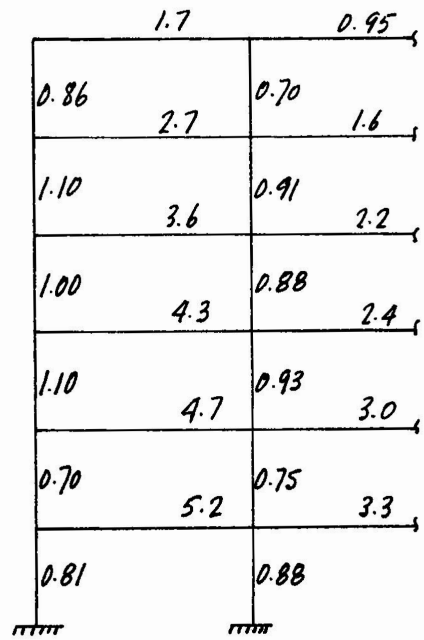


Fig. 4c. Nonlinear dynamic analysis: average damage ratios

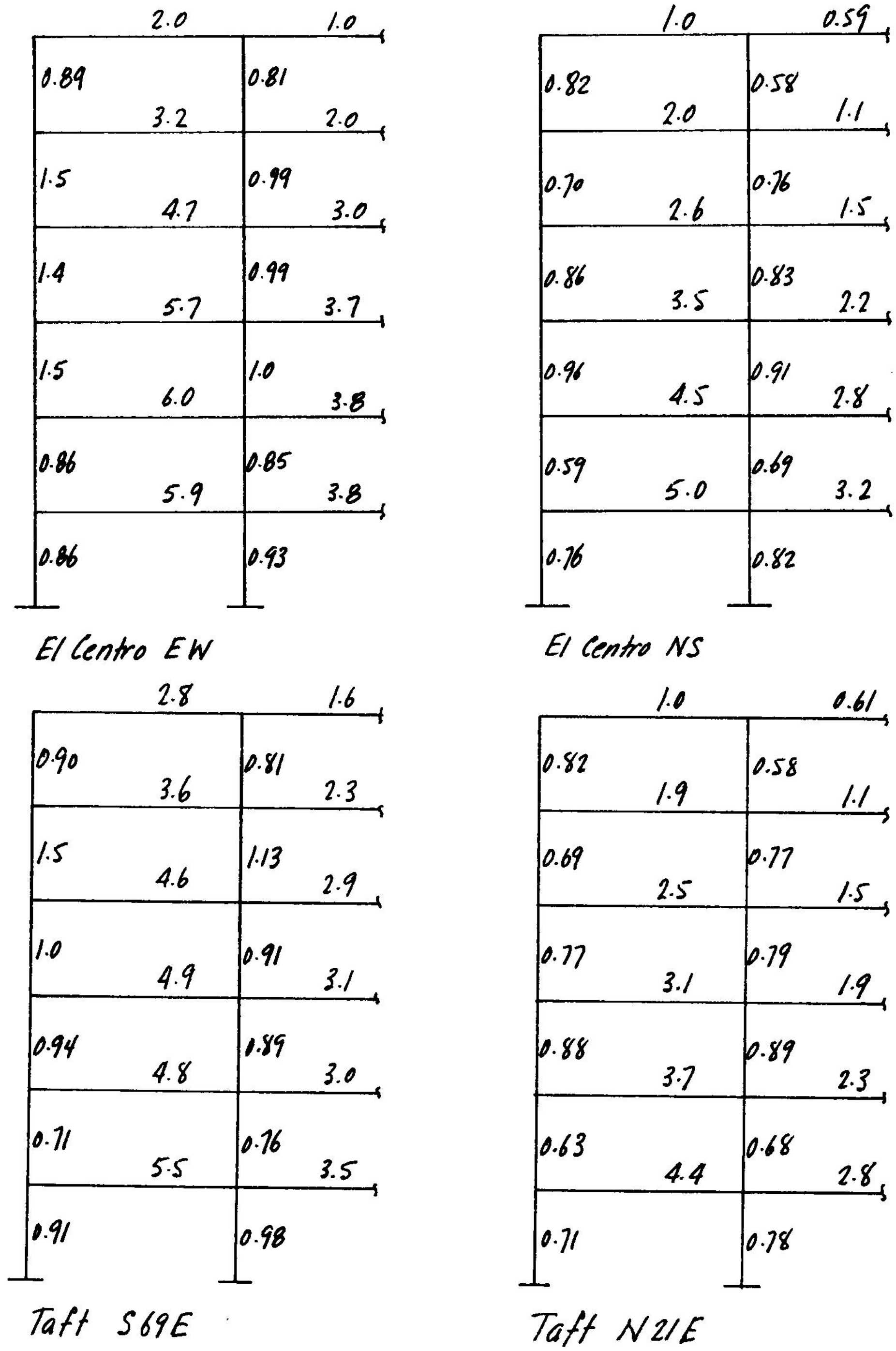
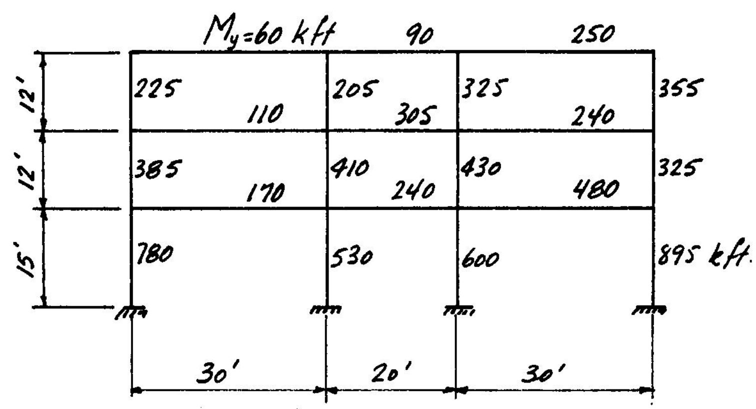


Fig. 4d. Nonlinear dynamic analysis: damage ratios for individual earthquakes.



W = 180 k.

W = 200k.

W = 240k.

	Level	Size	I
Exterior Columns	1	24" x 24"	13800 in^4
	2	24 x 24	13800 *
	3	21 x 21	8100
Interior Columns	1	21" x 21"	8100
	2	21 x 21	8100
	3	18 x 18	4375
Beams	1	18" x 24"	6910
	2	18 x 21	4630
	3	18 x 18	2920

* $I = 4375\ in^4$ for right side column.

$E = 3600$ ksi.

Fig. 5a. 3-bay, 3-storey frame: layout, member properties, weight.

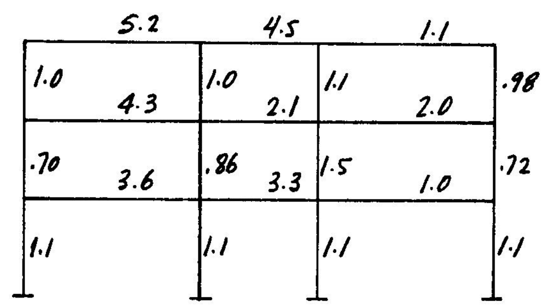


Fig. 5b. Modified substitute structure analysis: damage ratios

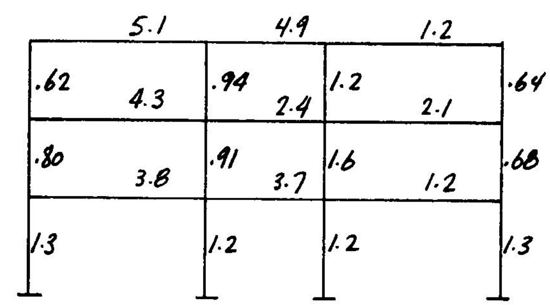


Fig. 5c. Nonlinear dynamic analysis: average damage ratios

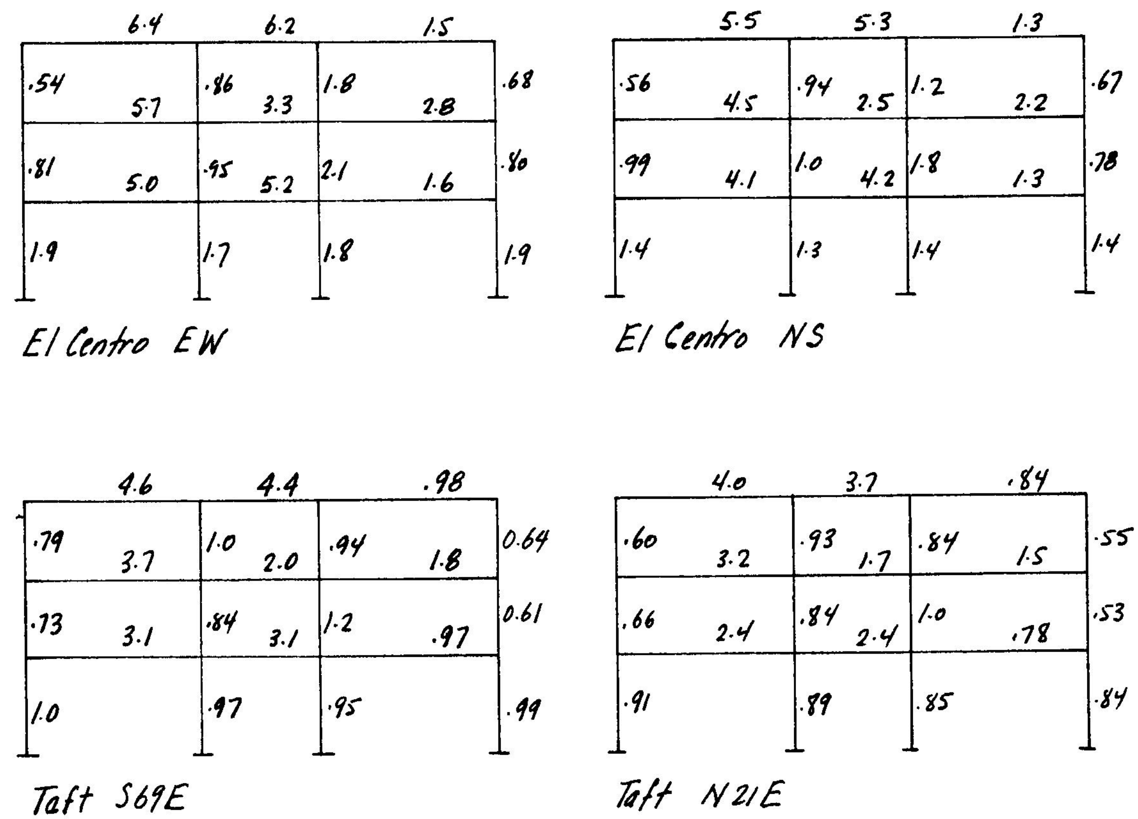
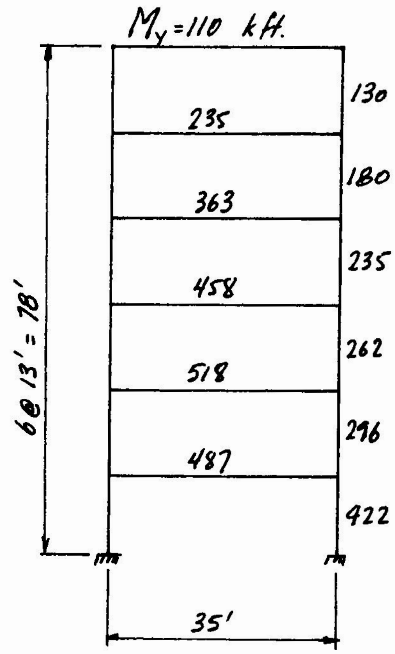


Fig. 5d. Nonlinear dynamic analysis: damage ratios for individual earthquakes.



	Level	Size	I
Columns	1	21" x 30"	24000 in^4
	2	" "	"
	3	21" x 27"	19200
	4	" "	"
	5	21" x 21"	9400
	6	" "	"
Beams	1	15" x 36"	33700
	2	" "	"
	3	" "	"
	4	" "	"
	5	" "	"
	6	15" x 31"	11600 in^4

E = 4320 ksi.

Weight = 90^k for roof,
= 110 k all other floors.

Fig. 6a. 1-bay, 6-storey frame: layout, member properties, weight.

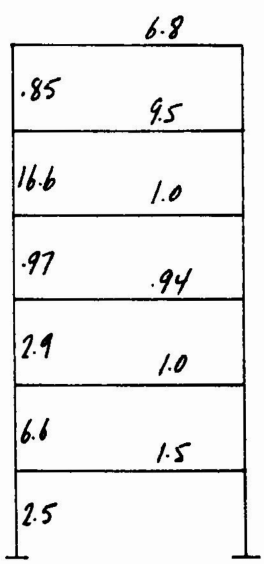


Fig. 6b. Modified substitute structure analysis: damage ratios.

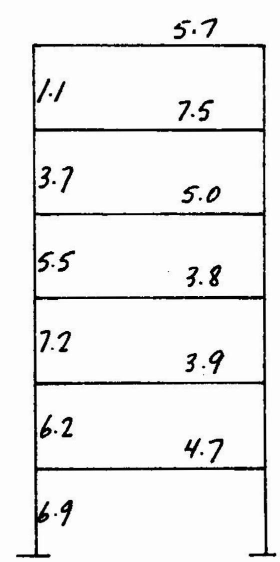


Fig. 6c. Nonlinear dynamic analysis: average damage ratios.

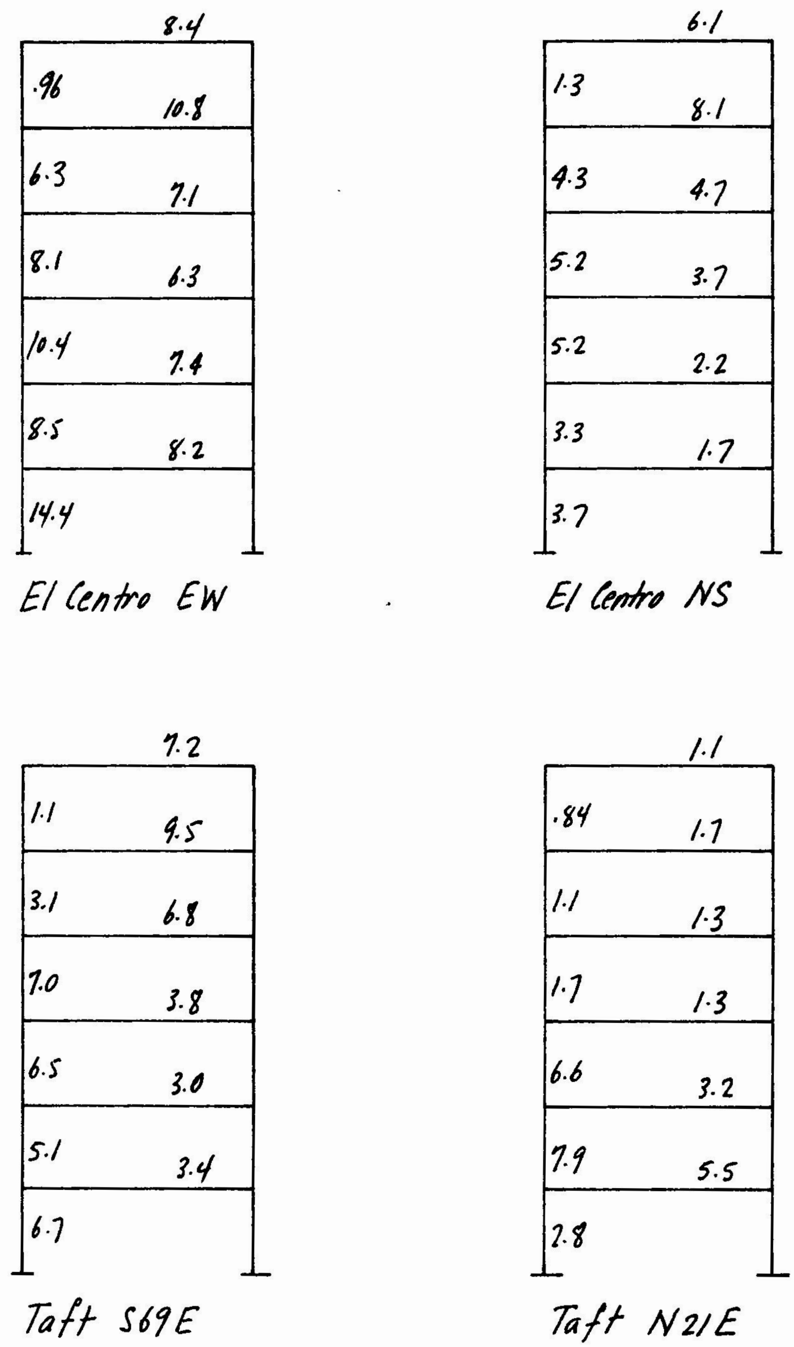


Fig. 6d. Nonlinear dynamic analysis: damage ratios for individual earthquakes.